

## Introduction

Radar beam broadening is an important tool which allows power to be concentrated in a larger main beam which wastes less energy and provides continuous coverage of a wider angular extent. We explore the use of subarrayed arrays, which are cheaper to manufacture than arrays with individual element excitation modification but are more difficult to synthesize into well-shaped patterns. Due to the high dimensionality of the problem, computational brute force techniques are infeasible for evaluating the entire space. We develop an effective cost function to evaluate generated radar patterns, and we optimize the pattern using various metaheuristic global optimization techniques to search for optimal phase values which have the lowest cost.

## Model

The normalized radar pattern  $p$  in decibels is generated by taking the FFT of the complex excitations of each element. This is evaluated by the following cost function  $C$ , which utilizes a comparison pattern  $m(d)$  where  $d$  is distance in degrees from the pattern center. The comparison pattern can be described as:

$$m = \begin{cases} 0 & d \leq \text{Beamwidth} \\ SLL & d > \text{Beamwidth} \end{cases}$$

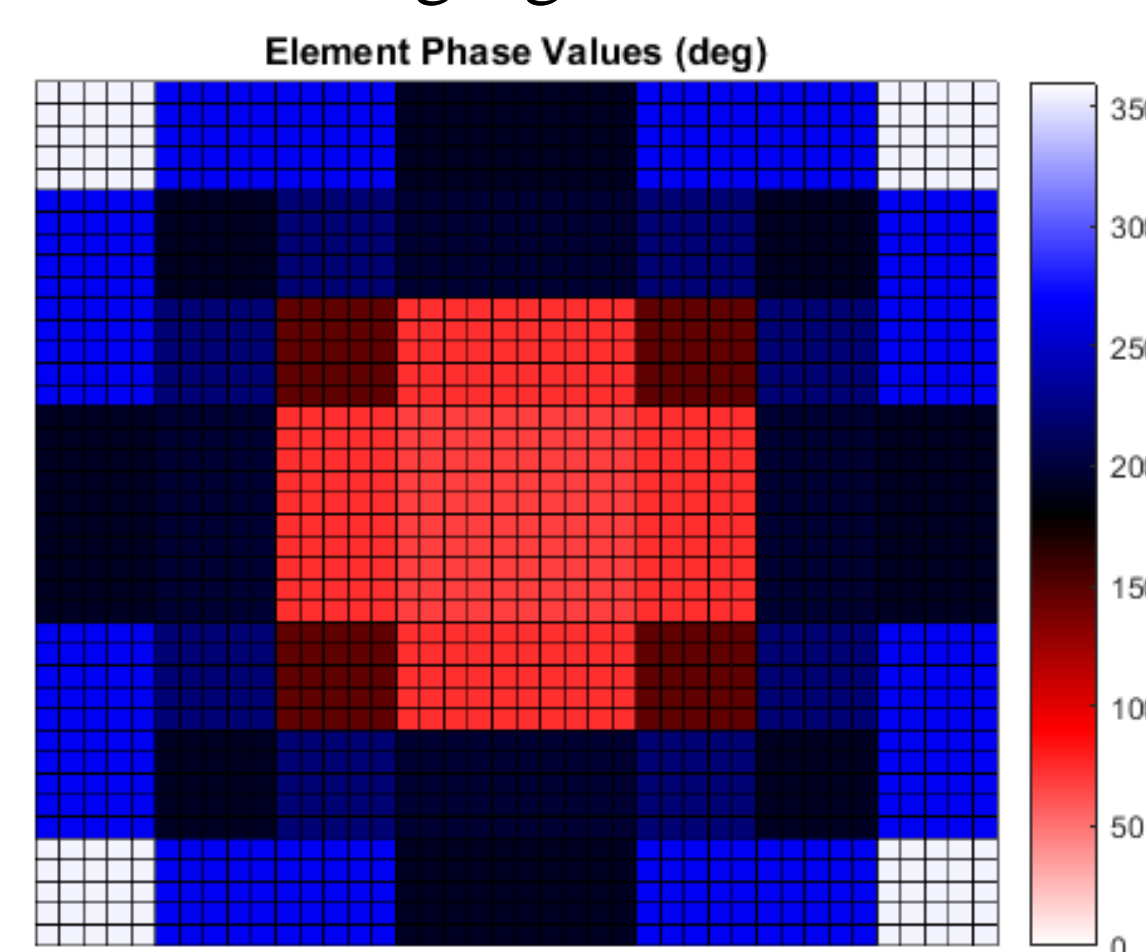
where  $SLL$  is the desired sidelobe level. The cost function is evaluated using an integral of the difference of the generated pattern from the desired pattern, excluding sufficiently low sidelobes but constricting large sidelobes with constant  $k$ .

$$r = \begin{cases} m - p & d \leq \text{Beamwidth} \\ k * \max(0, p - m) & d > \text{Beamwidth} \end{cases}$$

$$C = \log \int_p r$$

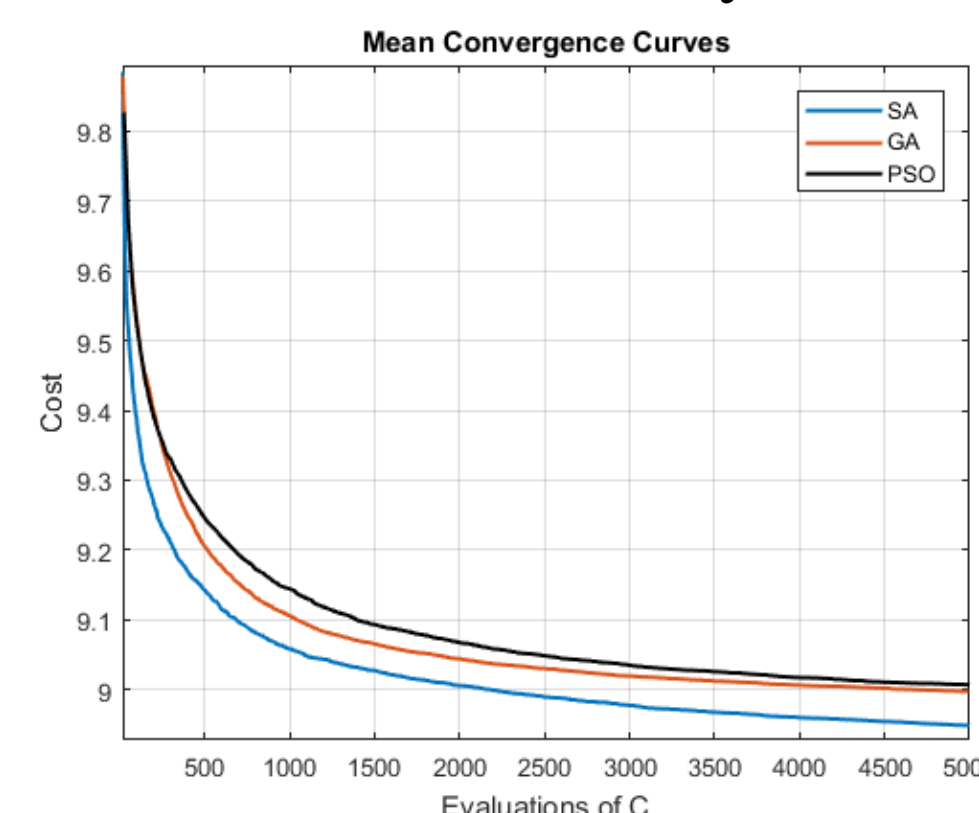
## Model Continued

A 40x40 element array uniformly spaced at half wavelength distance was used. It consists of 8x8 subarrays each comprising 5x5 elements and has 6-bit phase shifters. Algorithms used are simulated annealing (SA) [1] which simulates the behavior of cooling metal, particle swarm optimization (PSO) [2] which emulates particles physically moving towards the best solution, and a genetic algorithm (GA) [3] with elitism which emulates the evolutionary nature of genetics. In order to reduce the dimensionality of the problem, symmetry was utilized by assigning subarrays equidistant from the center the same phase. In this array, the dimensionality is reduced from 64 subarrays down to 9 independent variables. This can be seen in the following figure.



## Results

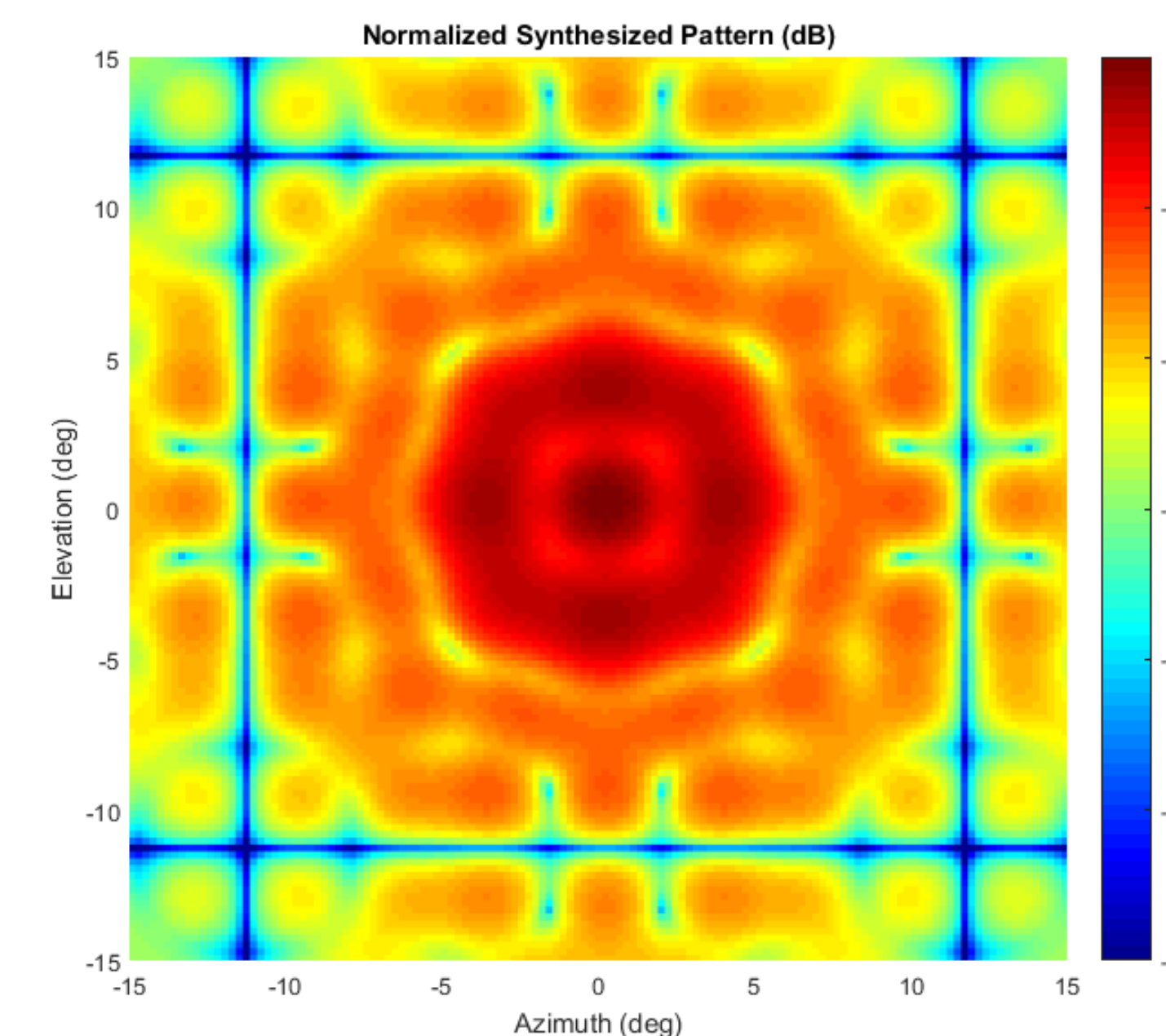
In order to effectively evaluate the stochastic algorithms, we record their best solution for each evaluation of the cost function as convergence curves. We record this data for each algorithm 200 times. This allows us to avoid the stochasticity of the resulting algorithm.



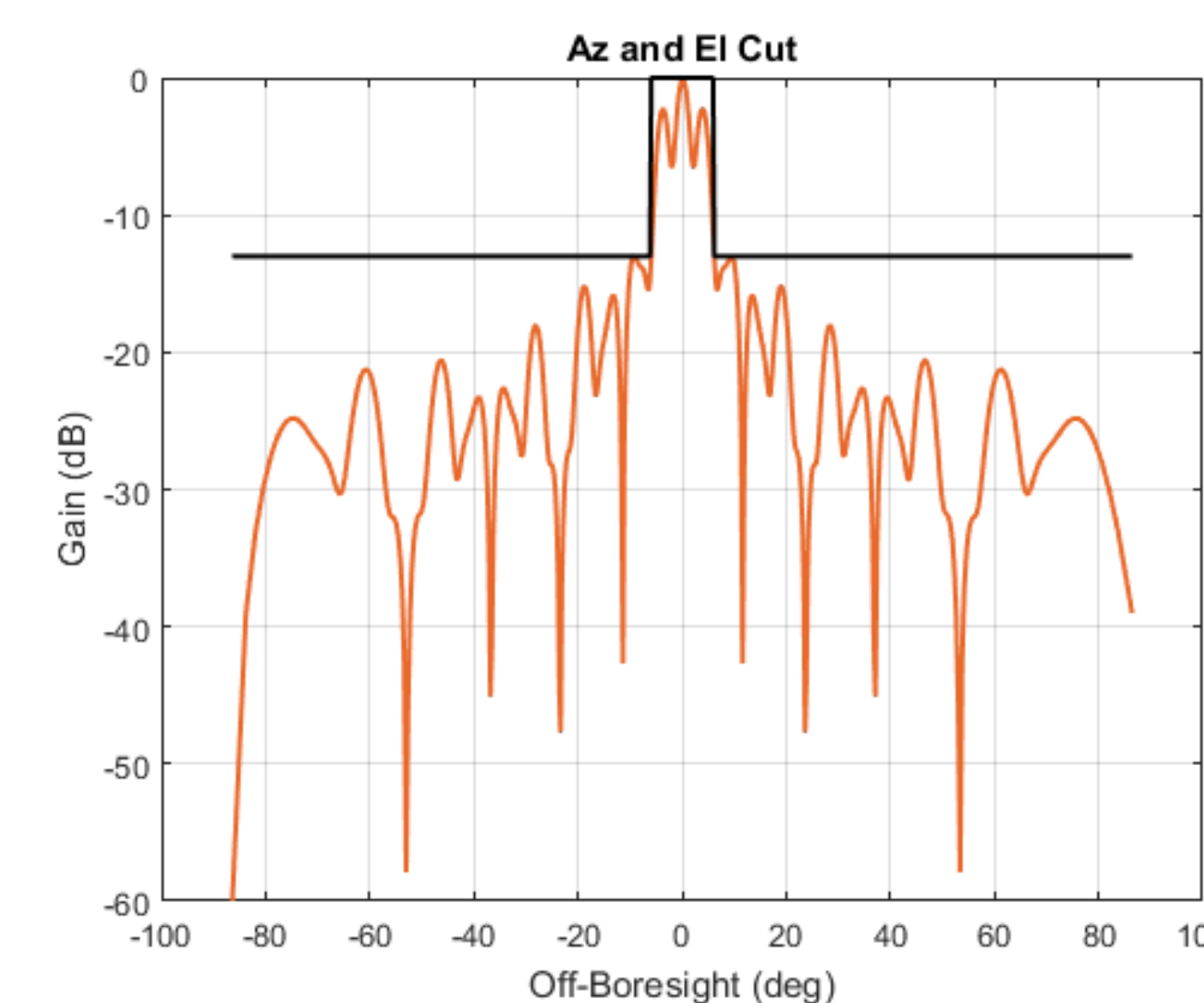
The above figure is the mean value of the convergence curve for each algorithm. It shows that on average, SA was more efficient and produced better solutions in the same amount of evaluations.

## Results Continued

The SA algorithm had 96% of its runs end with a fitness value less than 9. The GA had 60% and the PSO only had 49%. Simulated annealing tended to have smaller improvements, but they occurred more consistently, which is probably due to the single solution the algorithm remembers at a time. Although simulated annealing was more consistent, the best solutions from the runs of all the algorithms resulted in similar cost solutions that had similar phase values. In these solutions, phase values tended to increase as distance from the center of the radar increased.



The pattern shown here is created by SA is the best solution obtained. Sidelobes are constrained to the desired level of -13 dB and a broadened beam of 12 degrees is obtained. For this particular radar, this is likely very close to the best pattern which can be found.



## Conclusion

We explore global optimization techniques to achieve beam broadening in subarrayed arrays using phase-only modification of element excitations. All utilized techniques found solutions within one percent of each other in regards to power in the main beam. We find that simulated annealing is both the most efficient and most consistent, followed by genetic algorithms and particle swarm.

## Future Work

Future studies can examine these techniques for larger subarrayed arrays to better understand their effectiveness. Comparisons of these techniques utilized for both subarrayed and standard radars, with and without symmetry, is another potential path for research. Utilizing a supercomputing cluster could allow verification of the optimality of solutions found from these techniques.

## References

- [1] S. S. Skiena, *The algorithm design manual*. Springer Science & Business Media, 1998, vol. 1.
- [2] Y. Shi et al., "Particle swarm optimization: developments, applications and resources," in *evolutionary computation, 2001. Proceedings of the 2001 Congress on*, vol. 1. IEEE, 2001, pp. 81–86.
- [3] T. M. Mitchell, *Machine learning*. McGraw-Hill Boston, MA., 1997.

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